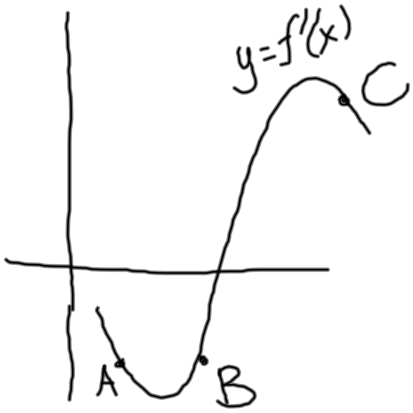
4)

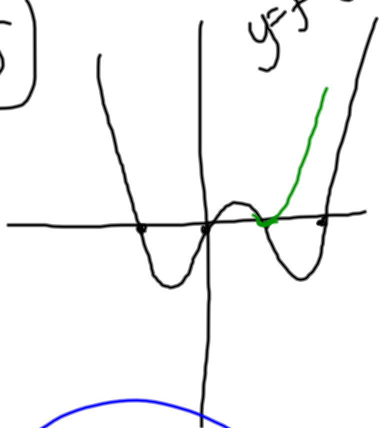


Find the signs of $f'(x)$, $f''(x)$ ~~at~~ at A, B, C

	$\frac{dy}{dx}$	$\frac{d^2y}{dx^2}$
A	−	−
B	−	+
C	+	−

9a
13
11a
5
4

5)



x-coordinates of all inflection pts

$f'' > 0 \Rightarrow$ c-up
$f'' < 0 \Rightarrow$ c-dn

Inf pt: concavity chgs

$x = -1$
$= 0$
$= 1$
$= 2$

13) $f(x) = (x+2)^3$

$f'(x) = 3(x+2)^2$

$f'(x) = 0 \Rightarrow x = -2$

sign of f': + + + + + + + + + + (at −2, f' = 0)

inc: $(-\infty, \infty)$

dec: nowhere

$f''(x) = 6(x+2)$

$f''(x) = 0 \Rightarrow x = -2$

c-up: $(-2, \infty)$

c-dn: $(-\infty, -2)$

inf pt: $x = -2$

or $(-2, 0)$

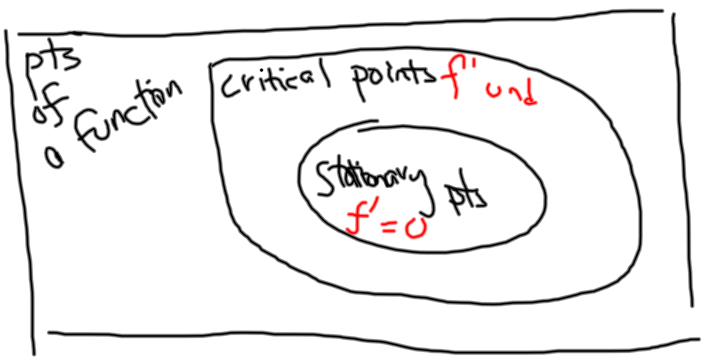
sign of $f''(x)$: − − − − (test −4) | −2, $f''=0$ | + + + + + + (test 0)

$\underbrace{\quad -4 \quad}_{\text{c-dn}}$ $\underbrace{\quad 0 \quad}_{\text{c-up}}$

A stationary point is a point on the graph (p.301)

at which $f'(x) = 0$.

So:



17
21
22

17) $f(x) = \dfrac{x^2}{x^2+2}$

$$f'(x) = \frac{(2x)(x^2+2) - (x^2)(2x)}{(x^2+2)^2}$$

$$= \frac{2x(2)}{(x^2+2)^2} = \frac{4x}{(x^2+2)^2}$$

critical pts

$f'(x) = 0$

$4x = 0$

$\Rightarrow x = 0$

f' undefined?

$(x^2+2)^2 = 0$

$\Rightarrow f'(x)$ always defined

sign of $f'(x)$: $- - - - - \overset{0}{|} + + + + + + + +$ (at 0)

increasing: $[0, \infty)$

decreasing: $(-\infty, 0]$

relative minimum @ $x = 0$

$f'(x) = \dfrac{4x}{(x^2+2)^2}$

$$f''(x) = \frac{(4)(x^2+2)^2 - 4x(2(x^2+2)(2x))}{((x^2+2)^2)^2}$$

$$f''(x) = \frac{4(x^2+2)[(x^2+2) - (4x^2)]}{(x^2+2)^4} = \frac{4(x^2+2)[2-3x^2]}{(x^2+2)^4}$$

$$= \frac{4[2-3x^2]}{(x^2+2)^3}$$

potential pts of inflection

$f'' = 0$

$4[2-3x^2] = 0$

$2 = 3x^2$

$\frac{2}{3} = x^2$

or $x = \pm\sqrt{\frac{2}{3}}$

f'' undefined

never

$\dfrac{4[2-3x^2]}{(x^2+2)^3}$

sign of f'': $- - - - - \overset{0}{|} + + + + + + + \overset{0}{|} - - - - -$

$-10 \quad -\sqrt{\frac{2}{3}} \quad 0 \quad +\sqrt{\frac{2}{3}} \quad 10$

c-up: $\left(-\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}\right)$

c-dn: $\left(-\infty, -\sqrt{\frac{2}{3}}\right) \cup \left(\sqrt{\frac{2}{3}}, \infty\right)$

pt of inf: @ $-\sqrt{\frac{2}{3}}, +\sqrt{\frac{2}{3}}$

$I_1 \cup I_2$ means any x in I_1 OR I_2 — union

$I_1 \cap I_2$ intersection — any x in both I_1 & I_2

21) $f(x) = x^{1/3}(x+4)$

$f'(x) = \frac{1}{3}(x^{-2/3})(x+4) + x^{1/3}$

$\frac{1}{3}(x^{1/3} + 4x^{-2/3}) + x^{1/3}$

$1/3x^{1/3} + 4/3x^{-2/3} + x^{1/3}$

$4/3x^{1/3} + 4/3x^{-2/3} = 0$

so $\frac{x^{1/3}}{x^{-2/3}}$

$\frac{4}{3}(x^{-\frac{2}{3}})(x+1) = 0$

critical pts

$f' = 0$, $x = -1$; f' und, $x = 0$

$x^{1/3} = -x^{-2/3}$

$\sqrt[3]{x} = -\sqrt[3]{x^2}$

c.p.t $x = -x^2$ → $x = -1$

$x + x^2 = 0$

$x(1+x) = 0$

sign of f': $- - - - - \overset{0}{|} + + + \overset{und}{|} + + + + +$ at -10, -1, $-\frac{1}{2}$, 0, 10

increasing: $[-1, \infty)$

decreasing: $(-\infty, -1]$

$f'(x) = 4/3x^{1/3} + 4/3x^{-2/3}$

$f''(x) = 4/9x^{-2/3} + \frac{-8}{9}x^{-5/3}$

$\frac{4}{9}x^{-5/3}(x-2) = 0 \Leftarrow 4/9x^{-2/3} + -8/9x^{-5/3} = 0$

$f'' = 0$, $x = 2$; f'' und, $x = 0$

$4/9x^{-2/3} = 8/9x^{-5/3}$

$x^{-2/3} = 1/2x^{-5/3}$

$\sqrt[3]{x^{-2}} = 1/2\sqrt[3]{x^{-5}}$

sign of f'': $+ + + \,|\, - - - \,|\, + + + +$ at 0, 1, 2, 10

$x^{-2} = 1/2x^{-5}$

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5.1 homework

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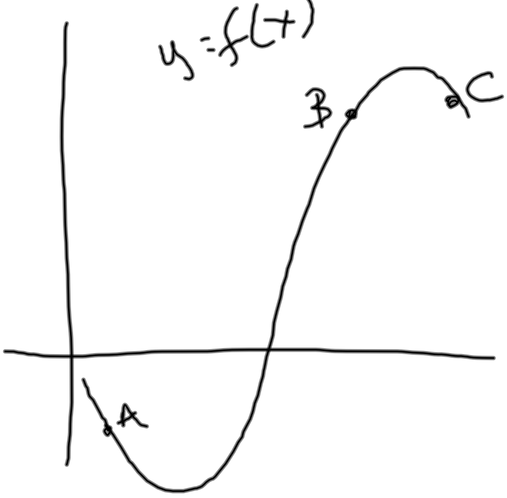
3
4
13

(3)



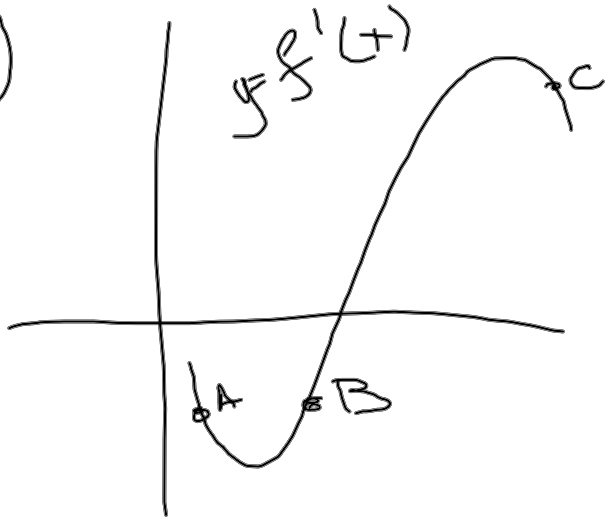
signs

	$\frac{dy}{dx}$	$\frac{d^2y}{dx^2}$
A	−	+
B	+	−
C	−	−

first deriv. properties: inc, dec, max, min

Second deriv prop: c-up, c-dn, inf pts.

(4)



Sign:

	$\frac{dy}{dx}$	$\frac{d^2y}{dx^2}$
A	−	−
B	−	+
C	+	−

13) $f(x) = (x+2)^3$

$f'(x) = 3(x+2)^2$

$f'(x) = 0 \Rightarrow \dots\ x = -2$

sign of $f'(x)$: $+ \ + \ + \ + \ + \ | \ + \ + \ + \ + \ +$ ...

$-10 \qquad -2 \qquad 0$

$f(x)$ is increasing $(-\infty, \infty)$

$f''(x) = 6(x+2)^0$

$f''(x) = 0 \Rightarrow x = -2$

sign of $f''(x)$: $- \ - \ - \ - \ - \ | \ + \ + \ + \ + \ + \ +$

$-1000 \qquad -2 \qquad \pi$

$\underbrace{\qquad\qquad}_{\text{c-dn}} \qquad \underbrace{\qquad\qquad}_{\text{c-up}}$

16
21
17- 18

16) $f(x) = x^4 - 8x^2 + 16$

$f'(x) = 4x^3 - 16x$

$f''(x) = 12x^2 - 16 = 4(3x^2 - 4)$

find potential points of inflection

$f''(x) = 0$

$4(3x^2-4) = 0$

$x^2 = \frac{4}{3}$

$x = \pm \frac{2}{\sqrt{3}}$

$f''(x)$ undefined?

nowhere

sign of $f'' = 4(3x^2-4)$

+ + + + + 0 - - - - - - 0 + + + + +

$-10 \quad -\frac{2}{\sqrt{3}} \quad 0 \quad +\frac{2}{\sqrt{3}} \quad 10$

c-up: $\left(-\infty, -\frac{2}{\sqrt{3}}\right) \cup \left(\frac{2}{\sqrt{3}}, \infty\right)$

c-dn: $\left(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)$

pt of inflection @ $x = -\frac{2}{\sqrt{3}}, +\frac{2}{\sqrt{3}}$

$\cup$ "union"

x in $I_1 \cup I_2$ if x is in I_1 OR x is in I_2

$\cap$ "intersection"

x in $I_1 \cap I_2$ if x is in I_1 AND x in I_2

5.1/17 $f(x) = \dfrac{x^2}{x^2+2}$

$$f'(x) = \frac{(2x)(x^2+2)-(x^2)(2x)}{(x^2+2)^2} = \frac{2x[(x^2+2)-x^2]}{(x^2+2)^2}$$

$$f'(x) = \frac{2x[2]}{(x^2+2)^2} = \frac{4x}{(x^2+2)^2}$$

find critical pts

$f'(x)=0$

$4x=0$

$\Rightarrow x=0$

$f'(x)$ undefined

$(x^2+2)^2=0$

nowhere

sign of $f' = \dfrac{4x}{(x^2+2)^2}$: $- - - - \overset{0}{|} + + + +$; test points -10, 0, 10

inc: $[0,\infty)$ relative minimum @ $x=0$

dec: $(-\infty,0]$

$f'(x) = \dfrac{4x}{(x^2+2)^2}$

$$f''(x) = \frac{(4)(x^2+2)^2-(4x)(2(x^2+2)(2x))}{((x^2+2)^2)^2}$$

$$f''(x) = \frac{4(x^2+2)[(x^2+2)-4x^2]}{(x^2+2)^4} = \frac{4(2-3x^2)}{(x^2+2)^3}$$

Find possible pts of inflection

$f''(x)=0$

$4(2-3x^2)=0$

$\Rightarrow 2-3x^2=0$

or $3x^2=2$

$x^2=\frac{2}{3}$ $x = \pm\sqrt{\frac{2}{3}}$

find $f''(x)$ undefined

none

sign of $f'' = \dfrac{4(2-3x^2)}{(x^2+2)^3}$: $- - - - \overset{0}{|} + + + + \overset{0}{|} - - -$; test points -10, $-\sqrt{\frac{2}{3}}$, 0, $\sqrt{\frac{2}{3}}$, 10

C-up: $\left(-\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}\right)$

C-dn: $\left(-\infty,-\sqrt{\frac{2}{3}}\right) \cup \left(\sqrt{\frac{2}{3}},\infty\right)$

pt of inflection @ $x = -\sqrt{\frac{2}{3}}, +\sqrt{\frac{2}{3}}$

5.1 21)

$f(x) = x^{1/3}(x+4)$

$f'(x) = \frac{1}{3}(x^{-2/3})(x+4) + (x^{1/3})$

$\frac{x^{1/3}}{x^{-2/3}}$

$= (x^{-2/3})\left[\frac{1}{3}(x+4) + x\right]$

$f'(x) = x^{-2/3}\left[\frac{4}{3}x + \frac{4}{3}\right]$

$x^{4/3} + 4x^{1/3}$

$\frac{4}{3}x^{1/3} + \frac{4}{3}x^{-2/3}$

$\frac{4}{3}(x^{1/3} + x^{-2/3})$

$\frac{4}{3}(x^{-2/3})(x+1)$

find crit pts

$f' = 0$

$\frac{4}{3}(x+1) = 0 \Rightarrow x = -1$

f' undefined

$x^{-2/3} = 0$

$\Rightarrow x = 0$

sign f': $- \; - \; - \; -$ | 0 | $+ \; + \; +$ | und | $+ \; + \; + \; +$

$x^{-2/3}\left[\frac{4}{3}(x+1)\right]$ -10 -1 $-\frac{1}{2}$ 0 10

inc: $[-1, \infty)$ rel min: @ $x = -1$

dec: $(-\infty, -1]$

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